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Nonlinear Realization of Partially Broken $N = 2$ Superconformal Symmetry in Four Dimensions

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Abstract

We investigate the nonlinear realization of spontaneously broken $N = 2$ superconformal symmetry in 4 dimensions. We particularly study Nambu-Goldstone degrees of freedom for the partial breaking of $N = 2$ superconformal symmetry down to $N = 1$ super-Poincaré symmetry, where we get the chiral NG multiplet of dilaton and the vector NG multiplet of NG fermion of broken Q-supersymmetry. Evaluating the covariant differentials and supervielbeins for the chiral as well as the full superspace, we obtain the nonlinear effective lagrangians.

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In the last few years there has been much interest in $N = 2$ supersymmetric gauge theory in the context of duality which provides us with the understanding of non-perturbative properties of supersymmetric field theories as well as string theories.

Hughes, Liu and Polchinski [1] pointed out that $N = 2$ global supersymmetry can be partially broken down to $N = 1$ supersymmetry by giving the argument on the way how to evade the existing no-go theorem of partial breaking of extended supersymmetry based on the supersymmetry current algebra. They also explicitly constructed the four-dimensional supermembrane solution of the six-dimensional supersymmetric gauge theory, in which the second supersymmetry, in the equivalent four-dimensional $N = 2$ theory, is spontaneously broken and the partial breaking is realized. While, Antoniadis, Partouche and Taylor [2] introduced the electric and magnetic Fayet-Iliopoulos terms in the $N = 2$ gauge theory of abelian vector multiplet and have shown that there occurs spontaneous breaking of $N = 2$ to $N = 1$ supersymmetry. This partial breaking induced by the Fayet-Iliopoulos terms has also been obtained by taking the flat limit of the $N = 2$ supergravity theories [3].

Bagger and Galperin [4, 5] have studied the nonlinear realization of $N = 2$ supersymmetry partially broken down to $N = 1$ supersymmetry [6, 7] and obtained the Nambu-Goldstone multiplet both for the cases; chiral multiplet [4] and vector-multiplet [5], and discussed the nonlinear transformation laws as well as the low-energy effective lagrangians [8].

Here in this paper, we shall investigate the nonlinear realization of $N = 2$ extended superconformal symmetry in four-dimensions, which is realized for the case of vanishing β-function in the N=2 supersymmetric QCD. We study the spontaneously breaking of this symmetry down to $N = 1$ super-Poincaré symmetry. We identify the Nambu-Goldstone degrees of freedom corresponding to the broken charges. It turns out that there appear a vector multiplet of NG fermion of the broken second Q-supersymmetry as well as a chiral multiplet of dilaton, axion and dilatino which are associated with the broken dilatation, chiral $U(1)$ rotation and the first S-supersymmetry generators. We note that the NG fermion of the broken second S-supersymmetry can be expressed as the derivative of the true NG-fermion of the second Q-supersymmetry. We obtain the effective interaction for the system of the dilaton multiplet coupled to the NG-vector

multiplet.

Let us suppose a certain symmetry, characterized by a group G, which is spontaneously broken down to its subsymmetry given by a subgroup H. In such a case there appear Nambu-Goldstone (NG) fields that transform nonlinearly under G as the coordinates of the coset space G/H. In the framework of the nonlinear realization, we can construct low-energy effective lagrangians describing the interactions of massless NG particles. Based on the framework [9, 10, 11] for the nonlinear realization of space-time symmetries, which is the modification of those for the internal symmetry [12] we can investigate the spontaneously broken extended supersymmetry [13].

We now consider the 4-dimensional N=2 superconformal group usually denoted by $SU(2,2/2)$, the generators of which are those of conformal group: translation P_μ, Lorentz rotation $M_{\mu\nu}$, conformal boost K_μ, and dilatation D operators; together with Q-supersymmetry generators: $Q_{\alpha A}$, $\bar{Q}^A_{\dot{\alpha}}$ $(A=1,2)$; S-supersymmetry generators: $S^{\alpha A}$, $\bar{S}^{\dot{\alpha}}_A$ $(A=1,2)$ and $SU(2)$ generators, ${T_A}^B$, the chiral $U(1)_R$ charge A, and the $U(2)$ charge${B_A}^B \equiv {T_A}^B + \frac{1}{6}\delta^B_A A$. Some relevant commutation relations are the following:

$$
\begin{aligned}
&[P_\mu, K_\nu] = 2i(g_{\mu\nu}D - M_{\mu\nu}) \ , \ \{Q_{\alpha A}, \bar{Q}^B_{\dot{\alpha}}\} = 2\delta^B_A \sigma_{\mu\alpha\dot{\alpha}} P^\mu \ , \ \{S^A_\alpha, \bar{S}_{\dot{\alpha}B}\} = 2\delta^A_B \sigma_{\mu\alpha\dot{\alpha}} K^\mu \\
&[Q_{\alpha A}, D] = \frac{i}{2} Q_{\alpha A} \quad , \quad [Q_{\alpha A}, A] = \frac{3}{2} Q_{\alpha A} \quad , \quad [\bar{S}^{\dot{\alpha}}_A, D] = -\frac{i}{2} \bar{S}^{\dot{\alpha}}_A \quad , \quad [S^{\alpha A}, A] = -\frac{3}{2} S^{\alpha A} \\
&\{Q_{\alpha A}, S^{\beta B}\} = \delta^B_A [{(\sigma^{\mu\nu})_\alpha}^\beta M_{\mu\nu} - 2i\delta^\beta_\alpha D] - 4\delta^\beta_\alpha {B_A}^B \\
&[Q_{\alpha A}, K_\mu] = \sigma_{\mu\alpha\dot{\alpha}} \bar{S}^{\dot{\alpha}}_A \quad , \quad [\bar{S}^{\dot{\alpha}}_A, P_\mu] = \bar{\sigma}^{\dot{\alpha}\alpha}_\mu Q_{\alpha A} \\
&[Q_{\alpha A}, {T_B}^C] = \delta^C_A Q_{\alpha B} - \frac{1}{2}\delta^C_B Q_{\alpha A} \quad , \quad [\bar{S}^{\dot{\alpha}}_A, {T_B}^C] = \delta^C_A \bar{S}^{\dot{\alpha}}_B - \frac{1}{2}\delta^C_B \bar{S}^{\dot{\alpha}}_A \\
&[{T_A}^B, {T_C}^D] = \delta^D_A {T_C}^B - \delta^B_C {T_A}^D \qquad (A,B,C,D = 1,2)
\end{aligned}
\tag{1}
$$

The various breaking patterns of this symmetry are illustrated in Fig.1. The spontaneous breaking of the pattern E has been the subject discussed in the literatures [1, 2, 3, 4, 5]. Here in this paper, we shall investigate the breaking pattern B.

Now we note that the charge commutation relation between an unbroken charge and a broken charge gives another broken charge. This relation puts the constraints on NG particles. For example, when conformal group is spontaneously broken to Poincaré group, the NG field ϕ_μ associated with the conformal boost K_μ is not an independent NG degree of freedom, but is related to the dilaton σ, the NG particle for dilatation, as $\phi_\mu \sim \partial_\mu \sigma$. This relation is obtained through the use of one of the commutation

relations of conformal algebra

$$[P_\mu, K_\nu] = 2i(g_{\mu\nu}D - M_{\mu\nu}) \tag{2}$$

and the Jacobi identity for P_μ, K_ν and σ

$$[[P_\mu, K_\nu], \sigma] + [[K_\nu, \sigma], P_\mu] + [[\sigma, P_\mu], K_\nu] = 0. \tag{3}$$

The vacuum expectation value of (3) gives $\langle 0|[K_\nu, \partial_\mu\sigma]|0\rangle = -2g_{\mu\nu}\langle 0|[D, \sigma]|0\rangle \neq 0$.

First we consider the case where D=4 N=2 superconformal group is spontaneously broken down to N=1 superconformal group, i.e. the breaking A in Fig.1. In this case, we have the broken generators $S^{\alpha 2}(\bar{S}_2^{\dot\alpha})$, $Q_{\alpha 2}(\bar{Q}_{\dot\alpha})$, T and $\bar{T}$, while $B_1^1 = 3R/4$ and B_2^2 remain unbroken. Here R is the $U(1)_R$ for N=1 superconformal group.

The constraints on NG particles are obtained in the same manner. The following commutation relations:

$$[\bar{S}_2^{\dot\alpha}, P_\mu] = \bar\sigma_\mu^{\dot\alpha\alpha} Q_{\alpha 2}\ ,\quad [Q_{\alpha 1}, T] = Q_{\alpha 2}\ ,\ \{Q_{\alpha 1}, S^{\beta 2}\} = -4\delta_\alpha^\beta \bar{T} \tag{4}$$

give rise to the constraints on N=1 NG multiplets $\chi^\alpha(x,\theta,\bar\theta),\ v(x,\theta,\bar\theta)$ and $\psi_{\alpha 2}(x,\theta,\bar\theta)$ for $Q_{\alpha 2},\ T$ and $S^{\beta 2}$. The Jacobi identities lead to

$$\bar\psi^{2\dot\alpha} \sim \bar\sigma_\mu^{\dot\alpha\alpha}\partial^\mu\chi_\alpha\ ,\ v \sim \{Q_1^\alpha, \chi_\alpha\}\ ,\ \bar\psi^{2\dot\alpha} \sim [\bar{Q}_1^{\dot\alpha}, v] \tag{5}$$

These transformations (5) show that there is an independent N=1 NG multiplet $\chi^\alpha(x,\theta,\bar\theta)$ which contains component fields $\chi^\alpha,\ v$ and $\psi_{\alpha 2}$, moreover we can identify these fields with Abelian gaugino, D-term and the derivative of gaugino, respectively. Now we impose the chirality condition on $\chi^\alpha(x,\theta,\bar\theta)$. Namely, $\chi^\alpha(x,\theta,\bar\theta)$ is a vector multiplet with spin $\frac{1}{2}$ and 1 fields. The spin 1 field does not really correspond to a NG degree of freedom, but a superpartner of the NG fermion. For this reason, we may call this multiplet as NG-Maxwell multiplet.

From the argument based on the charge commutation relation (4), we expect that if Q-supersymmetry is spontaneously broken, then the corresponding S-supersymmetry is broken as well and the NG fermion corresponding to the broken S-supercharge is written as a derivative of the NG fermion corresponding to the broken Q-supercharge. We shall present a proof for this statement by using a spectral representation [14]. Here we use, for simplicity, the four-component Majorana fermion representation.

Let us introduce Q-supercurrent $J_{\mu\alpha}$, S-supercurrent $S_{\mu\alpha}$ and consider the spectral representation of the two-point function. According to the PCT invariance, Lorentz covariance, the spectral condition as well as parity invariance, we can write down the two-point function as

$$\begin{aligned}\langle 0|\{J_{\mu\alpha}(x),\bar{\chi}_\beta(y)\}|0\rangle &= \int_0^\infty dm^2[\rho_1(m^2)i\partial_\mu\delta_{\alpha\beta}+\rho_2(m^2)(\gamma_\mu)_{\alpha\beta}+\rho_3(m^2)i\partial^\nu(\sigma_{\mu\nu})_{\alpha\beta}] \\ &\quad \times i\Delta(x-y;m^2).\end{aligned} \tag{6}$$

where $\rho_i(m^2)$ $(i=1,2,3)$ are the spectral functions. Q-supercurrent conservation $\partial^\mu J_{\mu\alpha}=0$ as well as S-supercurrent conservation $(\gamma^\mu)_{\alpha\beta}J_{\mu\beta}=0$ lead to

$$m^2\rho_1(m^2)=0\ ,\ \ \rho_2(m^2)=0,\ \ \rho_3(m^2)=-\frac{1}{3}\rho_1(m^2) \tag{7}$$

Hence, we can write down the spectral representation of the two-point function as $(\rho_1(m^2)\equiv\rho(m^2))$:

$$\langle 0|\{J_{\mu\alpha}(x),\bar{\chi}_\beta(y)\}|0\rangle=\int_0^\infty dm^2\rho(m^2)[i\partial_\mu\delta_{\alpha\beta}-\frac{1}{3}i\partial^\nu(\sigma_{\mu\nu})_{\alpha\beta}]\ i\Delta(x-y;m^2), \tag{8}$$

with $m^2\rho(m^2)=0$, which implies

$$\rho(m^2)=c\delta(m^2). \tag{9}$$

We note that if $c\neq 0$, Q-SUSY is spontaneously broken and the corresponding NG fermion is χ_α.

Next we calculate the following vacuum expectation value of the equal-time commutation relation, by using $S_{\mu\alpha}(x)=x_\nu\gamma^\nu_{\alpha\beta}J_{\mu\beta}(x)$ as

$$\begin{aligned}&\langle 0|\{S_\alpha,\,\partial\!\!\!/_{\gamma\beta}\bar{\chi}_\gamma(y)\}|0\rangle|_{x_0=y_0} \\ =\ &\int d^4x\ \delta(x_0-y_0)x_\mu\gamma^\mu_{\alpha\delta}\ \partial\!\!\!/_{\gamma\beta}(y)\langle 0|\{J_{0\delta}(x),\bar{\chi}_\gamma(y)\}|0\rangle \\ =\ &4c\delta_{\alpha\beta}.\end{aligned}$$

Thus if Q-supersymmetry is spontaneously broken, then S-supersymmetry is spontaneously broken as well. And the NG fermion for the broken S-supercharge is written by the derivative of the NG fermion for the broken Q-supercharge.

We now turn to spontaneous breaking of D=4 N=2 superconformal group down to N=1 super-Poincaré group, i.e. the breaking pattern B in Fig.1. We derive the constraints imposed on NG particles as in the previous case. The commutation relations:

$$\begin{aligned} &\{Q_{\alpha 1}, S^{\beta 1}\} = (\sigma^{\mu\nu})_\alpha{}^\beta M_{\mu\nu} - 2i\delta_\alpha^\beta D - 4\delta_\alpha^\beta B_1{}^1 \\ &[Q_{\alpha 1}, K_\mu] = \frac{1}{2}\sigma_{\mu\alpha\dot\alpha}\bar S_1^{\dot\alpha}, \ [P_\mu, K_\nu] = 2i(g_{\mu\nu}D - M_{\mu\nu}) \end{aligned} \tag{10}$$

and (2,4) give constraints on N=1 NG multiplets. As before, Jacobi identities between (10) and NG particles lead to

$$\psi_{\alpha 1} \sim [Q_{\alpha 1}, \sigma + i\rho] \ , \ \phi_\mu \sim \bar\sigma_\mu^{\dot\alpha\alpha}\{Q_{\alpha 1}, \bar\psi_{\dot\alpha 1}\} \ , \ \phi_\mu \sim \partial_\mu \sigma. \tag{11}$$

These transformations (11) show that there exists another independent N=1 NG multiplet $(\sigma + i\rho)(x, \theta, \bar\theta) \equiv \phi\ (x, \theta, \bar\theta)$ which consists of component fields ϕ and $\psi_{\alpha 1}$. We put the chirality condition on $\phi\ (x, \theta, \bar\theta)$. Thus we conclude that $\phi\ (x, \theta, \bar\theta)$ is the NG-background chiral superfield which contains a complex spin 0 field $\phi = \sigma + i\rho$, where σ is the dilaton and ρ is the axion, and spin $\frac{1}{2}$ field $\psi_{\alpha 1}$ is the dilatino.

Now we turn to the coset construction for the nonlinear realization. When D=4 N=2 superconformal group is spontaneously broken to N=1 super-Poincaré group, the relevant left-invariant coset representative is given by

$$L(x,\ \theta,\ \bar\theta) = T\ F\ U \tag{12}$$

where

$$\begin{aligned} &T = \exp(ix\cdot P + i\theta^A Q_A + i\bar\theta_A \bar Q^A) \qquad\qquad \theta^2 \equiv \chi\ ,\ \bar\theta_2 \equiv \bar\chi \\ &F = \exp(i\phi\cdot K + i\psi_A S^A + i\bar\psi^A \bar S_A)\exp(i\sigma D + iv_1 B_1{}^1 + iv_2 B_2{}^2) \\ &U = \exp(ivT + i\bar v \bar T). \end{aligned} \tag{13}$$

Under the left multiplication of a group element $g \in G$, L transforms as

$$L \to gL = L'h \qquad (h \in H), \tag{14}$$

from which we obtain the transformation laws of the NG fields. We calculate the Cartan differential 1-form as

$$\begin{aligned} L^{-1}dL \ = \ & i\mathrm{D}x\cdot P + i\mathrm{D}\theta^A Q_A + i\mathrm{D}\bar\theta_A \bar Q^A + i\mathrm{D}\phi\cdot K + i\mathrm{D}\psi_A S^A + i\mathrm{D}\bar\psi^A \bar S_A \\ & + i\mathrm{D}\sigma D + i\mathrm{D}v_1 B_1{}^1 + i\mathrm{D}v_2 B_2{}^2 + i\mathrm{D}vT + i\mathrm{D}\bar v \bar T + \frac{1}{2}\omega_{\mu\nu}M^{\mu\nu} \end{aligned} \tag{15}$$

where

$$\begin{aligned}
\mathrm{D}x^\mu &= \exp(-\sigma)[dx^\mu + id\theta^A\sigma^\mu\bar{\theta}_A + id\bar{\theta}_A\bar{\sigma}^\mu\theta^A] \equiv \exp(-\sigma)dl^\mu \\
\mathrm{D}\theta^{\alpha C} &= W_A{}^B U_B{}^C[d\theta^{\alpha A} - i(\bar{\psi}^A\bar{\sigma}_\mu)^\alpha dl^\mu] \\
\mathrm{D}\bar{\theta}_{\dot{\alpha}C} &= (W^\dagger)_B{}^A(U^{-1})_C{}^B[d\bar{\theta}_{\dot{\alpha}A} - i(\psi_A\sigma_\mu)_{\dot{\alpha}}dl^\mu] \\
\mathrm{D}\sigma &= d\sigma - 2[dl\cdot\phi - d\theta^A\psi_A - d\bar{\theta}_A\bar{\psi}^A] \qquad (16)\\
\mathrm{D}\rho &= d\rho - 2[dl^\mu(\bar{\psi}^A\bar{\sigma}_\mu\psi_A) + id\theta^A\psi_A - id\bar{\theta}_A\bar{\psi}^A] \\
U_A{}^B &\equiv \exp i(v\tau + \bar{v}\bar{\tau})_A{}^B = \begin{pmatrix} \cos\sqrt{v\bar{v}} & i\sqrt{\frac{v}{\bar{v}}}\sin\sqrt{v\bar{v}} \\ i\sqrt{\frac{\bar{v}}{v}}\sin\sqrt{v\bar{v}} & \cos\sqrt{v\bar{v}} \end{pmatrix} \\
W_A{}^B &\equiv \exp\left[-\frac{1}{2}\{(\sigma - 3i\rho)1 - iv_3\tau_3\}\right]_A{}^B\ ,\ \rho \equiv \frac{1}{6}(v_1 + v_2)\ ,\ v_3 \equiv v_1 - v_2.
\end{aligned}$$

We take $Q_{2\alpha}$ to be the broken supercharge while keeping $Q_{1\alpha}$ unbroken. The NG field of the broken $Q_{2\alpha}$ is denoted by χ_α. Here one should also note that we only consider the case where $SU(2)\times U(1)$ symmetrey is broken to $U(1)$. Namely, the three generators; $T = B_2{}^1$, $\bar{T} = B_1{}^2$ and $A = 3(B_1{}^1 + B_2{}^2)$ are broken and $T_3 = \frac{1}{2}(B_1{}^1 - B_2{}^2)$ remains unbroken. This is consitent with the relations obtained by the charge algebras. Further, if T_3 were broken, we would need another real scalar as well as one fermionic superpartner, but there are no such particles in the present case. Therefore, it is reasonable to keep T_3 unbroken. This amounts to take $v_3 = 0$ and $W_A{}^B$ is reduced to $\exp[-\frac{1}{2}(\sigma - 3i\rho)]\ \delta_A^B$.

Now let us proceed to the constraints to be imposed on particles in the framework of nonlinear realization in order to eliminate the unphysical degrees of freedom. Before doing this, we introduce the superspace coordinates $X^A = (x^\mu, \theta^\alpha, \bar{\theta}_{\dot{\alpha}})$ and the supervielbein E_M^A defined as

$$\mathrm{D}X^A = dX^M E_M^A\ , \quad \mathrm{D}X^A \equiv (\mathrm{D}x^\mu,\ \mathrm{D}\theta^\alpha,\ \mathrm{D}\bar{\theta}_{\dot{\alpha}})\ ,\ dX^M \equiv (dx^m,\ d\theta^a,\ d\bar{\theta}_{\dot{a}}). \qquad (17)$$

In general the constraints should be invariant under the nonlinear transformations, and therefore its form is represented as $\mathrm{D}_A\xi =$ constant for any NG particles ξ, where

$$\mathrm{D}_A\xi = \frac{\mathrm{D}\xi}{\mathrm{D}X^A} = (E^{-1})_A{}^M\frac{\mathrm{D}\xi}{dX^M} \qquad (18)$$

is the covariant derivative of NG field in the nonlinear realization. This is because the covariant derivatives of NG particles transform linearly under the full group. In our

present case we set the following constraints:

$$
\begin{aligned}
&\mathrm{D}_\mu \chi^\alpha = \mathrm{D}_{\dot\alpha} \chi^\alpha = \mathrm{D}_\alpha \chi^\alpha = 0, \quad \mathrm{D}_\mu \bar\chi_{\dot\alpha} = \mathrm{D}_\alpha \bar\chi_{\dot\alpha} = \mathrm{D}^{\dot\alpha} \bar\chi_{\dot\alpha} = 0, \\
&\mathrm{D}_A \sigma = 0 \ (A = \mu, \alpha, \dot\alpha); \ \mathrm{D}_\alpha \rho = 0, \ \mathrm{D}_{\dot\alpha} \rho = 0.
\end{aligned} \tag{19}
$$

We will see that these constraints realize the relations (5,11) and further impose the chirality conditions on χ as well as on $\phi \equiv \sigma + i\rho$. Note that we do not require $\mathrm{D}_\mu \rho = 0$, because it would lead to an over-constraint among the independent NG fields. Moreover, other possible constraint like $\mathrm{D}_A \psi_2 = 0$ does not match the relations (5,11).

Using the supervielbein matrix elements, we define covariant derivatives in the NG background as ($\partial_m = \partial/\partial x^m$, $\partial_a = \partial/\partial\theta^a$, $\partial^{\dot a} = \partial/\partial\bar\theta_{\dot a}$):

$$
\mathcal{D}_\mu \equiv (e^{-1})_\mu{}^m \partial_m \ , \ \mathcal{D}_a \equiv \partial_a - e_a{}^\mu \mathcal{D}_\mu \ , \ \bar{\mathcal{D}}^{\dot a} \equiv \partial^{\dot a} - e^{\dot a \mu} \mathcal{D}_\mu . \tag{20}
$$

$$
\begin{aligned}
e_m{}^\mu &\equiv \delta_m^\mu + i(\partial_m \chi \sigma^\mu \bar\chi + \partial_m \bar\chi \bar\sigma^\mu \chi), \quad (e^{-1})_\mu{}^m e_m{}^\nu = \delta_\mu{}^\nu \\
e_a{}^\mu &\equiv i(\sigma^\mu \bar\theta)_a + i(\partial_a \chi \sigma^\mu \bar\chi + \partial_a \bar\chi \bar\sigma^\mu \chi) \\
e^{\dot a \mu} &\equiv i(\bar\sigma^\mu \theta)^{\dot a} + i(\partial^{\dot a} \chi \sigma^\mu \bar\chi + \partial^{\dot a} \bar\chi \bar\sigma^\mu \chi)
\end{aligned} \tag{21}
$$

which coincide with those introduced in [5]. Now we solve (19) and find

$$
\bar{\mathcal{D}}^{\dot\beta} \chi^\alpha = 0 \ , \ \bar{\mathcal{D}}^{\dot\beta} \phi = 0 \tag{22}
$$

$$
\bar\psi^1_{\dot\alpha} = -\frac{1}{4} \ (\bar Y^{-1})^{\dot\beta}_{\dot\alpha} [\ \bar{\mathcal{D}}_{\dot\beta} \bar\phi + i \mathcal{D}_\mu (\chi\sigma^\mu)_{\dot\gamma} \bar{\mathcal{D}}_{\dot\beta} \bar\chi^{\dot\gamma}], \quad \bar Y^{\dot\alpha}_{\dot\beta} \equiv \delta^{\dot\alpha}_{\dot\beta} + \frac{{U_1}^2}{{U_2}^2} \bar{\mathcal{D}}_{\dot\beta} \bar\chi^{\dot\alpha} \tag{23}
$$

$$
\bar\psi^2_{\dot\alpha} = -\frac{1}{4} \ (\bar Y^{-1})^{\dot\beta}_{\dot\alpha} [\ i\mathcal{D}_\mu (\chi\sigma^\mu)_{\dot\beta} - \frac{{U_1}^2}{{U_2}^2} \bar{\mathcal{D}}_{\dot\beta} \bar\phi] \tag{24}
$$

$$
i\sqrt{\frac{v}{\bar v}} \tan 2\sqrt{v \bar v} = (\mathcal{D}\chi + \frac{1}{2} \bar{\mathcal{D}} \bar\chi \mathcal{D}_\alpha \chi_\beta \mathcal{D}^\alpha \chi^\beta) \ (1 - \frac{1}{4} \mathcal{D}_\gamma \chi_\delta \mathcal{D}^\gamma \chi^\delta \bar{\mathcal{D}}_{\dot\alpha} \bar\chi_{\dot\beta} \bar{\mathcal{D}}^{\dot\alpha} \bar\chi^{\dot\beta}) \tag{25}
$$

$$
\phi_\mu = \frac{1}{2} \mathcal{D}_\mu \sigma + (\mathcal{D}_\mu \chi) \psi_2 + (\mathcal{D}_\mu \bar\chi) \bar\psi^2 \tag{26}
$$

Note that the conjugated relations also hold for (22) -(25). The two equations of (22) are the chirality conditions for χ^α and $\phi = \sigma + i\rho$. From (23), the superfield of the dilatino ψ_1, the superpartner of the dilaton, is given by the other superfields. The equation (24) gives the superfield ψ_2 as derivatives of $\bar\chi$ and ϕ, in accordance with the statement we made before by using the charge algebras and the spectral representation.

The equation (25) shows that the NG field of the broken SU(2) generators, T and $\bar{T}$, is given by the superfield χ. The last equation (26) indicates that ϕ_μ is not an independent NG degree of freedom but is given by other NG fields.

The explicit expression for the supervielbein in the full superspace is given by

$$E_M{}^A = \begin{pmatrix} A_m{}^\mu & D_m{}^\alpha & G_{m\dot{\alpha}} \\ B_a{}^\mu & E_a{}^\alpha & H_{a\dot{\alpha}} \\ C^{\dot{a}\mu} & F^{\dot{a}\alpha} & J^{\dot{a}}{}_{\dot{\alpha}} \end{pmatrix} \tag{27}$$

where

$$\begin{aligned}
&A_m{}^\mu = e^{-\sigma} e_m{}^\mu, \quad B_a{}^\mu = e^{-\sigma} e_a{}^\mu, \quad C^{\dot{a}\mu} = e^{-\sigma} e^{\dot{a}\mu} \\
&D_m{}^\alpha = e^{-\frac{1}{2}(\sigma - 3i\rho)} \left[\partial_m \chi^\alpha U_2^1 - i e_m{}^\mu (\bar{\psi}^A \bar{\sigma}_\mu)^\alpha U_A^1 \right] \\
&E_a{}^\alpha = e^{-\frac{1}{2}(\sigma - 3i\rho)} \left[U_1^1 \delta_a^\alpha + U_2^1 \partial_a \chi^\alpha - i e_a^\mu (\bar{\psi}^A \bar{\sigma}_\mu)^\alpha U_A^1 \right] \\
&F^{\dot{a}\alpha} = e^{-\frac{1}{2}(\sigma - 3i\rho)} \left[U_2^1 \partial^{\dot{a}} \chi^\alpha - i e^{\dot{a}\mu} (\bar{\psi}^A \bar{\sigma}_\mu) U_A^1 \right] \\
&G_{m\dot{\alpha}} = e^{-\frac{1}{2}(\sigma + 3i\rho)} \left[\partial_m \bar{\chi}_{\dot{\alpha}} (U^{-1})_1^2 - i e_m^\mu (\psi_A \sigma_\mu)_{\dot{\alpha}} (U^{-1})_1^A \right] \\
&H_{a\dot{\alpha}} = e^{-\frac{1}{2}(\sigma + 3i\rho)} \left[(U^{-1})_1^2 \partial_a \bar{\chi}_{\dot{\alpha}} - i e_a^\mu (\psi_A \sigma_\mu)_{\dot{\alpha}} (U^{-1})_1^A \right] \\
&J^{\dot{a}}{}_{\dot{\alpha}} = e^{-\frac{1}{2}(\sigma + 3i\rho)} \left[(U^{-1})_1^1 \delta_{\dot{\alpha}}^{\dot{a}} + (U^{-1})_1^2 \partial^{\dot{a}} \bar{\chi}_{\dot{\alpha}} - i e^{\dot{a}\mu} (\psi_A \sigma_\mu)_{\dot{\alpha}} (U^{-1})_1^A \right]
\end{aligned} \tag{28}$$

From these result, we can compute the superdeterminant of E_M^A as

$$\begin{aligned}
\text{sdet } (E_M^A) &= \det \, (A_m{}^\mu) \cdot \det{}^{-1} (\hat{D} - \hat{B} A^{-1} \hat{C}) \\
&= e^{-2\sigma} \det \, (e_m{}^\mu) \left| 1 - \frac{i}{2} \mathcal{D}\chi \sqrt{\frac{\bar{v}}{v}} \tan \sqrt{v\bar{v}} \right|^{-2}
\end{aligned} \tag{29}$$

where we have denoted the submatrices as

$$\begin{aligned}
&E_M^A = \begin{pmatrix} A_m{}^\mu & \hat{C} \\ \hat{B} & \hat{D} \end{pmatrix}, \qquad \hat{B} = \begin{pmatrix} B_a{}^\mu \\ C^{\dot{a}\mu} \end{pmatrix} \\
&\hat{C} = \begin{pmatrix} D_m{}^\alpha & G_{m\dot{\alpha}} \end{pmatrix}, \qquad \hat{D} = \begin{pmatrix} E_a{}^\alpha & H_{a\dot{\alpha}} \\ F^{\dot{a}\alpha} & J^{\dot{a}}{}_{\dot{\alpha}} \end{pmatrix}.
\end{aligned} \tag{30}$$

The inverse of the supervielbein, $(E^{-1})_A{}^M$ is computed in a similar way.

The simplest invariant action for the dilaton multiplet is obtained from the above superdeterminant, by forming the invariant phase volume.

$$\begin{aligned}
&\int d^4x d^2\theta d^2\bar{\theta} \, \text{sdet} E_M^A \\
= &\int d^4x \mathcal{L}_D + \cdots
\end{aligned} \tag{31}$$

where the nonlinear lagrangian $\mathcal{L}_D$ turns out to be

$$\mathcal{L}_D = e^{-\varphi-\varphi^*}\Big[-\partial^m\varphi^*\partial_m\varphi + \frac{i}{2}\partial_m\bar{\psi}\bar{\sigma}^m\psi - \frac{i}{2}\bar{\psi}\bar{\sigma}^m\partial_m\psi - \frac{i}{2}(\partial_m\varphi - \partial_m\varphi^*)\bar{\psi}\bar{\sigma}^m\psi + (F^* + \frac{1}{2}\bar{\psi}^2)(F + \frac{1}{2}\psi^2)\Big] \tag{32}$$

where we denote the first, second and auxiliary components of the superfield ϕ by φ, ψ and F, respectively. The ψ is, at the same time, the first component of the superfield ψ_1. This is the same effective lagrangian for the sponatneous breaking of N=1 superconformal to N=1 super-Poincaré symmetry corresponding to the pattern D in Fig.1 and discussed in ref.[15].

Now we introduce the chiral superspace in the NG background [5] , $(x_L{}^m, \theta^\alpha)$, as

$$x_L{}^m = x^m - i\theta\sigma^m\bar{\theta} - i\chi\sigma^m\bar{\chi} \tag{33}$$

The supervielbein $E_{L}{}_M^A$ for the chiral superspace in NG background is

$$\begin{pmatrix} Dx^\mu, & D\theta^\alpha \end{pmatrix} = \begin{pmatrix} dx_L{}^m, & d\theta^a \end{pmatrix}\begin{pmatrix} A_m{}^\mu & C_m{}^\alpha \\ B_a{}^\mu & D_a{}^\alpha \end{pmatrix} \equiv \begin{pmatrix} dx_L{}^m, & d\theta^a \end{pmatrix} E_L \tag{34}$$

where

$$\begin{aligned} A_m{}^\mu &= e^{-\sigma}e^\mu_{Lm}, \quad e^\mu_{Lm} = \delta^\mu_m + 2i\partial^L_m\chi\sigma^\mu\bar{\chi} \\ B^\mu_a &= e^{-\sigma}e^\mu_{La}, \quad e^\mu_{La} = 2i(\sigma^\mu\bar{\theta})_a + 2i\partial_a\chi\sigma^\mu\bar{\chi} \\ C^\alpha_m &= e^{\frac{1}{2}(\sigma-3i\rho)}\left(\partial^L_m\chi^\alpha U^1_2 - ie^\mu_{Lm}(\bar{\psi}^A\bar{\sigma}_\mu)^\alpha U^1_A\right) \\ D^\alpha_a &= e^{\frac{1}{2}(\sigma-3i\rho)}\left(\delta^\alpha_a U^1_1 + \partial_a\chi^\alpha U^1_2 - ie^\mu_{La}(\bar{\psi}^A\bar{\sigma}_\mu)^\alpha U^1_A\right) \end{aligned} \tag{35}$$

The chiral superdeterminant is given by

$$\mathrm{sdet}E_L = \det A\cdot\det{}^{-1}(D - BA^{-1}C) = e^{-3\sigma-3i\rho}\det(e^\mu_{Lm})[1 - \frac{i}{2}\sqrt{\frac{\bar{v}}{v}}\tan\sqrt{v\bar{v}}\mathcal{D}\chi]^{-1}. \tag{36}$$

The invariant nonlinear lagarangian has the following form for the chiral superspace:

$$\int d^4x_L d^2\theta\ \mathrm{sdet}E_L\ f(D_A\xi, \Phi) + \mathrm{h.c.} \tag{37}$$

where ξ is any NG field and Φ is a spectator field which transforms linearly in the nonlinear realization.

Now we construct the invariant lagrangian for the vector multiplet of NG fermion for the broken supercharge Q_2.

Under the supersymmetry transformation of Q_2: $g = \exp i(\eta Q_2 + \bar{\eta}\bar{Q}_2)$,

$$\begin{aligned} &x^m \to x'^m = x^m + i(\eta\sigma^m\bar{\chi} - \chi\sigma^m\bar{\eta}) \\ &\theta^\alpha \to \theta'^\alpha = \theta^\alpha, \quad \bar{\theta}_{\dot{\alpha}} \to \bar{\theta}'_{\dot{\alpha}} = \bar{\theta}_{\dot{\alpha}} \end{aligned} \tag{38}$$

and the NG fermion varies as

$$\chi'^\alpha(x', \theta', \bar{\theta}') = \chi^\alpha(x, \theta, \bar{\theta}) + \eta^\alpha, \quad \bar{\chi}'_{\dot{\alpha}}(x', \theta', \bar{\theta}') = \bar{\chi}_{\dot{\alpha}}(x, \theta, \bar{\theta}) + \bar{\eta}_{\dot{\alpha}} \tag{39}$$

Note that the NG fermion field χ_α starts with $W_\alpha = i\bar{D}^2 D_\alpha V$,

$$\chi_\alpha = W_\alpha + \frac{1}{4}\bar{D}^2(\bar{W}^2)W_\alpha - iW\sigma^\mu\bar{W}\partial_\mu W_\alpha + \mathcal{O}(W^5). \tag{40}$$

where $D_\alpha = \partial_\alpha - i(\sigma^\mu\bar{\theta})_\alpha\partial_\mu$ and $\bar{D}^{\dot{\alpha}} = \partial^{\dot{\alpha}} - i(\sigma^\mu\theta)^{\dot{\alpha}}\partial_\mu$ are the ordinary supercovariant derivative and $V(x, \theta, \bar{\theta})$ is the vector superfield [5].

The invariant effective lagrangian for the vector multiplet can be obtained by the standard prescription (37) in the nonlinear realization. We take the function f to be

$$f(\mathrm{D}_A\xi) = \frac{1}{3}\left[1 + \frac{1}{2}\mathrm{D}_{(\dot{\alpha}}\bar{\chi}_{\dot{\beta})}\mathrm{D}^{(\dot{\alpha}}\bar{\chi}^{\dot{\beta})}\right] \tag{41}$$

where

$$\mathrm{D}_{(\dot{\alpha}}\bar{\chi}_{\dot{\beta})} = \frac{1}{2}(\mathrm{D}_{\dot{\alpha}}\bar{\chi}_{\dot{\beta}} + \mathrm{D}_{\dot{\beta}}\bar{\chi}_{\dot{\alpha}}) \tag{42}$$

is the symmetric part and is equal to

$$\mathrm{D}_{(\dot{\alpha}}\bar{\chi}_{\dot{\beta})} = \frac{(U^{-1})_2{}^2}{\bar{\Delta}}\bar{\mathcal{D}}_{(\dot{\alpha}}\bar{\chi}_{\dot{\beta})} = \bar{\mathcal{D}}_{(\dot{\alpha}}\bar{\chi}_{\dot{\beta})} + \cdots \tag{43}$$

with

$$\bar{\Delta} = 1 + \frac{i}{2}\sqrt{\frac{v}{\bar{v}}}\tan\sqrt{v\bar{v}}\bar{D}\bar{\chi} \tag{44}$$

The effective lagrangian for the vector multiplet (or NG-Maxwell multiplet) arises from the interplay of the D-term (31) and the F-term given as follows,

$$\begin{aligned} &\int d^4x_L d^2\theta\ \frac{1}{3}e^{-3\phi}\det(e^\mu_{Lm})[1 - \frac{i}{2}\sqrt{\frac{\bar{v}}{v}}\tan\sqrt{v\bar{v}}\mathcal{D}\chi]^{-1}[1 + \frac{1}{2}\mathrm{D}_{(\dot{\alpha}}\bar{\chi}_{\dot{\beta})}\mathrm{D}^{(\dot{\alpha}}\bar{\chi}^{\dot{\beta})}] + \text{h.c.} \\ &= \int d^4x_L d^2\theta e^{-3\phi}\ (1 + 2i\partial^L_m\chi\sigma^m\bar{\chi} + \cdots)(1 - \frac{1}{4}\mathcal{D}\chi\bar{\mathcal{D}}\bar{\chi} + \cdots) \\ &\qquad \times(1 - \frac{1}{4}(\bar{\mathcal{D}}\bar{\chi})^2 + \frac{1}{2}\bar{\mathcal{D}}_{\dot{\alpha}}\bar{\chi}_{\dot{\beta}}\bar{\mathcal{D}}^{\dot{\alpha}}\bar{\chi}^{\dot{\beta}}) + \text{h.c.} \\ &= \int d^4x_L \mathcal{L}_{NGM} \end{aligned} \tag{45}$$

where the following relation has been used:

$$\bar{\mathcal{D}}_{(\dot{\alpha}}\bar{\chi}_{\dot{\beta})}\bar{\mathcal{D}}^{(\dot{\alpha}}\bar{\chi}^{\dot{\beta})} = \bar{\mathcal{D}}_{\dot{\alpha}}\bar{\chi}_{\dot{\beta}}\bar{\mathcal{D}}^{\dot{\alpha}}\bar{\chi}^{\dot{\beta}} - \frac{1}{2}(\bar{\mathcal{D}}\bar{\chi})^2. \tag{46}$$

To the order of $\bar{\chi}^2 = \bar{W}^2 + \cdots$ we get

$$\mathcal{L}_{NGM} = \int d^2\theta\ e^{-3\phi}\ \frac{1}{3}(1 - \frac{1}{4}\bar{D}^2(\bar{W}^2) + \cdots) + \text{h.c.} \tag{47}$$

where the integrand turns out to be chiral to this order. In this sense the choice of $f(\mathrm{D}_A\xi)$ is unique to the order of $\bar{\chi}^2$. Note that we have used the Bianchi identity, $DW = -\bar{D}\bar{W}$. The total lagrangian

$$\mathcal{L} = \mathcal{L}_D + \mathcal{L}_{NGM} \tag{48}$$

provides the kinetic term for the vector multiplet by using the equation of motion for F and F^*. Hence we get

$$-\int d^4x e^{-4\sigma}\left\{1 + \frac{1}{4}F_{mn}F^{mn} - \frac{i}{2}\partial_m\bar{\lambda}\sigma^m\lambda + \frac{i}{2}\bar{\lambda}\bar{\sigma}^m\partial_m\lambda - \frac{1}{2}D^2 + \mathcal{O}(F_{mn}^4)\right\} \tag{49}$$

This action is invariant under the nonlinear scale transformation: $x^\mu \to e^\kappa x^\mu$, where the dilaton transforms as $\sigma \to \sigma + \kappa$ and the gauge field strength F_{mn} as well as the auxiliary field D have vanishing scale dimensions and λ has a scale dimension 1/2. This is because the superfield χ has a scale dimension 1/2, as can be seen from the coset construction.

Here we should note the connection with the breaking pattern C, which was studied in ref. [16]. In that case there appear a $N = 2$ multiplet consisting of dilaton σ, axion ρ, dilatinos ψ_i, $\bar{\psi}^i (i = 1, 2)$ and vector gauge field A_μ. The effective lagrangian in that case is similar to the sum of (32) and (49). If the $N = 2$ supersymmetry breaks down to $N = 1$, this multiplet splits into two $N = 1$ multiplets; the vector multiplet and the chiral multiplet.

Finally, a comment on the Born-Infeld action [17, 18] is in order. It is not straightforward to extend the Born-Infeld action for the partially broken $N = 2$ super-Poincaré symmetry discussed by Bagger and Galperin [5] to the present case, where the dilaton multiplet is coupled to the NG-Maxwell multiplet. It might be possible to derive the Born-Infeld action by summing up the higher order powers of F_{mn} in (49), which is now under investigation.

References

[1] J. Hughes, J. Liu and J. Polchinski, *Phys. Lett.* **B180** (1986) 370;
J. Hughes and J. Polchinski, *Nucl. Phys.* **B278** (1986) 147.

[2] I. Antoniadis, H. Partuche and T. R. Taylor, *Phys. Lett.* **B372** (1996) 83;
I. Antoniadis and T. R. Taylor, CPTH-PC445.0496, hep-th/9604062(1996);
H. Partouche and B. Pioline, CPTH-PC496.0297, hep-th/9702115 (1997).

[3] S. Ferrara, L. Girardello and M. Poratti *Phys. Lett.* **B376** (1996) 275;
M. Poratti,hep-th/9609073 (1996);
P. Fré, L. Girardello, I. Pensando and M. Trigiante, hep-th/9611188 (1996).

[4] J. Bagger and A. Galperin, *Phys. Lett.* **B336** (1994) 25.

[5] J. Bagger and A. Galperin, *Phys. Rev.* **D55** (1997) 1091.

[6] J. Bagger and J. Wess, *Phys. Lett.* **138B** (1984) 105.

[7] S. Samuel and J. Wess, *Nucl. Phys.* **B221** (1983) 153.

[8] J. Bagger, *Nucl. Phys.* **B** (Proc. Suppl.)**52A** (1997) 362.

[9] D. V. Volkov, *Sov. J. Particles and Nuclei* **4** (1973) 3.

[10] V. I. Ogievetsky, in Proc. of 10th Winter School of Theoretical Physics in Karpacz, Wroclaw (1974) vol.1, p.117.

[11] T. Uematsu and C. K. Zachos, *Nucl. Phys.* **B201** (1982) 250.

[12] S. Coleman, J. Wess and B. Zumino, *Phys. Rev.* **177** (1969) 2239;
C. Callan, S. Coleman, J. Wess and B. Zumino, *Phys. Rev.* **177** (1969) 2247.

[13] P. Fayet, *Nucl. Phys.* **B149** (1979) 137.

[14] K. Higashijima, Proc. XX Int. Colloquium on Group Theoretical Methods in Physics, July 4-9, 1994 Toyonaka, Japan (World Scientific, 1995), pp. 223-228.

[15] K. Kobayashi and T. Uematsu, *Nucl. Phys.* **B263** (1986) 309.

[16] K. Kobayashi, K. H. Lee and T. Uematsu, *Nucl. Phys.* **B309** (1988) 669.

[17] M. Born and L. Infeld, *Proc. Roy. Soc. London* **A144** (1934) 425.

[18] S. Cecotti and S. Ferrara, *Phys. Lett.* **B187** (1986) 335.

Figure Caption

Fig.1 Various patterns for spontaneous breaking of $N = 2$ superconformal symmetry